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CROSS-CURRICULAR LINKAGE OF MATHEMATICS AND TECHNOLOGY WITH APPLICATIONS IN GRAPHIC TECHNOLOGY

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It is necessary to introduce a new methodical approach to the teaching of mathematics at the faculties of technology which needs to be directed towards mathematical modelling of real problems appearing in technology and economy. This approach to the teaching of mathematics has for years now been successfully implemented in the higher education system in the United States of America. This paper describes the way to link the content of mathematics courses with the content of courses within graphic technology. The described approach could also be successfully implemented in all other faculties of technology. This would contribute to raising the level of mathematics education and students would, after graduation, be better trained to apply mathematical knowledge in practice. The new way of educating would raise the level of mathematical competences of students and would make them more competitive at the increasingly more demanding labour market.
Key words: education, mathematics, technology, cross-curricular linkage.

Međupredmetno povezivanje matematike i tehnologije s primjenama u grafičkoj tehnologiji. Sve više se uočava potreba za uvođenjem novog metodičkog pristupa u nastavu matematikena tehnološkim fakultetima, koja treba biti usmjerena prema matematičkom modeliranju realnih problema koji se pojavljuju u tehnologiji i gospodarstvu. Ovaj pristup nastavi matematike godinama se uspješno provodi u visokoškolskom obrazovnom sustavu SAD-a. U radu je opisan način na koji bi se trebali povezati sadržaji matematičkih kolegija sa sadržajima kolegija iz grafičke tehnologije. Navedeni pristup mogao bi se uspješno primijeniti i na sve ostale tehnološke fakultete. To bi pridonijelo podizanju razine matematičke edukacije te bi studenti nakon završetka studija bili kvalitetnije osposobljeni za primjenu matematičkih znanja u praksi. Novi način edukacije podigao bi razinu matematičkih kompetencija studenata te ih učinio konkurentnijim na sve zahtjevnijem tržištu rada.
Ključne riječi: edukacija, matematika, tehnologija, međupredmetno povezivanje.

INTRODUCTION

Mathematical knowledge is today the basic driving force of the development of new technologies in all areas of science and industry, especially in the most developed countries such as the USA, Japan and the countries of the European Union [1], [2], [3], [4].

This trend motivated by the fast development and application of mathematical knowledge in all areas of technology [2], which today makes an important part of the American educational system, should find its more important place also in Croatian faculties of technology.

Currently in Croatian faculties of technology mathematics courses offer a very broad basis needed to understand the content of a large part of basic and professional

courses in all study programmes [7]. Besides mastering the universal mathematical language and acquiring basic mathematical literacy, the purpose of mathematics courses is also to train students to recognize and solve specific problems connected with specific features of technologies which they study at their respective educational institutions, faculties, polytechnics, etc. This aim can only be achieved with the interdisciplinary cooperation [8], [9] between teachers of mathematics and teachers and scientists from other areas.

The new approach to the teaching of mathematics described in this paper is similar to guidelines to be found at the level of secondary education in the new national framework curriculum for elementary and secondary education [10] which is trying to link the ideas and principles regulated by the European Union [11], [12]. The activities in line with the above-mentioned guidelines are also expected from the University in Zagreb so a more comparative and more coherent higher education system could be created which would be compatible with the European system [13].

As an example mathematics programmes at the American universities can be cited which are accompanied with the corresponding standardized literature, the so-called calculuses which usually include the study of elementary functions and differ-rential and integral calculus. This is also a basic part of Croatian mathematics programmes at the faculties of technology. At the same time the calculuses are modulated according to specific features of respective courses they are taught at. The calculuses contain a large number of examples consisting of simplified real problems from certain areas. Hence the calculus for engineers [14] is different in content from the calculus for the students of economics or the students of biomedicine [16].

Teachers with year-long experience in teaching calculus at the American universities understand differently its importance within the higher education system at the faculties they teach. So W. Dale Compton sees the importance of calculus in training students to work with complex linear and nonlinear systems specific for engineering approach. According to him, students should be directed towards creating their own critical opinion and evaluation of problems which demand numerical approach that often implies non-exact approximative solutions [17].

Donald B. Small advocates the initiative for an interdisciplinary cooperation [18] which motivated the National Research Council (NRC) to push the faculties towards joint creation of courses and workshops which illustrate the links between different disciplines. Interdisciplinary efforts shall influence the development of students' competences to learn, when faced with real problems specific for their area of study, to see those problems from a different, broader perspective. Because the real problems faced by workers in the real world of labour cannot be limited to only one discipline.

M. S. Townend and S. S. Sazhin share the opinion that the approach which puts the emphasis on mere mastering of different calculus techniques should be abandoned and that the real balance between the application of mathematics and deep understanding of mathematical content should be found [19], [20]. In other words, it is necessary to carefully balance the relation between theory in the teaching of mathematics and its application in engineering practice. The problem seen by Townend and Sazhin is also present at the Croatian faculties of technology.

According to the stated opinions it is necessary to reorganize the teaching of mathematics at the Croatian faculties of technology. To be more specific, at the

Faculty of Graphic Arts the mathematics courses should find their motivation in the specific problems from graphic technology. In this way the emphasis within the planned content of mathematics course would be put to those lessons more important for the students of the Faculty of Graphic Arts.

NEW METHODICAL APPROACH TO THE TEACHING OF MATHEMATICS AT THE FACULTIES OF TECHNOLOGY

The teaching of mathematics at the faculties of technology is focused on studying theoretical basics of mathematics. Abstract mathematical theory is applied to geometry problems. This develops specific mathematical competences related to the development of abstract thinking. However, this approach neglects the development of general mathematical competences. In other words, students are not trained enough for mathematical modelling of real problems and for solving such problems with the help of mathematical instruments.

All this leads to a gap between the implementation of the curriculum and students' expectations that basic courses such as mathematics should provide support for the kind of knowledge specific for the study they chose. Furthermore, the teachers of mathematics believe that other teachers will use mathematics according to the content of their own courses and hence do not believe that the teaching of mathematics should be adapted in advance to other non-mathematical courses. In this way the teaching of mathematics remains within the field of mathematics which is understandable given that the aim is to gain mathematical knowledge. However, this way of teaching mathematics does not offer students enough motivation to study mathematics in more details because they fail to see its importance for their area of study.

Hence for example the teaching of mathematics at the faculty of Graphic Arts should be enriched with specific examples from graphic technology and design which cannot be solved without using mathematical modelling. In such cases a standard order should be followed which implies the motivation through setting and defining a real problem from the practice. Further steps thereafter would be to apply an appropriate mathematical theory which includes the mathematical description and the definition of the problem and to translate the real problem into mathematical language.

When the problem is correctly formulated in mathematical terms, mathematical instruments are introduced needed to find the solution of the problem. Formal mathematical theory includes the introduction of mathematical definitions and proving certain theorems, propositions, lemmas, etc. The mathematical instrument should then be applied to the given problem in order to find its mathematical solution. At the end the mathematical solution should be appropriately linked with the starting real problem where the interpretation of the mathematical solution gets the meaning. This procedure can be graphically presented in the following chart (Figure 1).

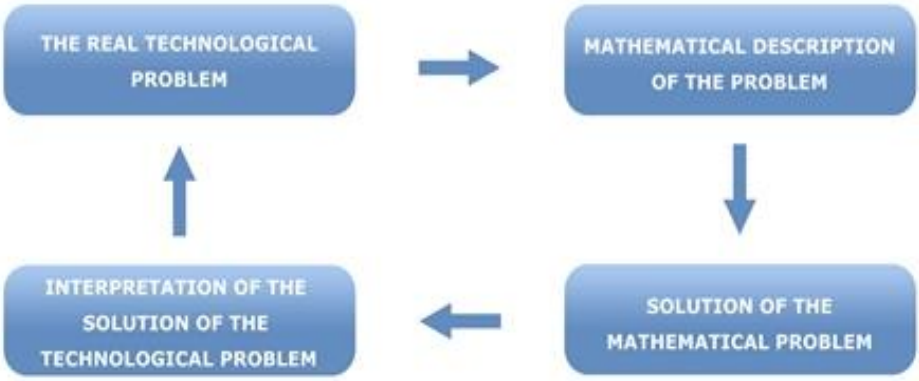

Figure 1. Mathematical description of the real technological problem, its interpretation and solution
Slika 1. Matematički opis realnog tehnološkog problema, njegova interpretacija i rješenje

It can be expected that this new approach to the teaching of mathematics would have many positive effects on students' education:

(a) the increase of students' interest for the content of mathematical courses at the faculties of technology;
(b) the increase of students' capabilities to perceive and model specific problems related to respective technologies;
(c) the increase of capabilities of engineering way of thinking;
(d) the introduction of motivational examples from practice to the teaching of mathematics would contribute to the development of students' positive attitude towards mathematics;
(e) higher level of students' capabilities to analyze, synthesize, design, compare and give critical arguments via an interdisciplinary approach.

Interdisciplinary linking of mathematical and technological courses would increase the competitiveness of the students graduating from the faculties of technology at the labour market in Croatia but also in other countries of the European Union. This broad discussion which goes beyond the scope of a paper requires a wider discussion and a joint cooperation between mathematicians and scientists from other areas. The largest contribution to this discussion is expected from those teachers whose courses contain content which cannot be appropriately understood without using mathematical instruments. It is also necessary to include students into this discussion in order to see their expectations what leaves room to further research.

AN EXAMPLE OF A LESSON ON GAUSSIAN ELIMINATION METHOD WITH THE APPLICATION TO THE PROBLEM OF PACKAGING GRAPHIC PRODUCTS

This chapter gives an exemplary planning of the lesson entitled "Gaussian elimination method with the application to the problem of packaging" which is taught at the Faculty of Graphic Arts. This lesson uses the interdisciplinary approach which contains the combination of mathematical and graphic technology knowledge. What is foreseen apart from the standard mathematical knowledge is also the analysis of the problem of packaging graphic products which occurs in graphic companies, for example Kraš. This lesson was created to stimulate students' critical thinking and detailed processing of information what would enable them to link the new mathematical knowledge to the already acquired knowledge and skills from graphic technology.

Table 1. Planning the lesson "Gaussian elimination method with the application to the problem of packaging"
Tablica 1. Plan izvođenja lekcije „Gaussova metoda eliminacijes primjenom na problem pakiranja"

Lesson: Gaussian elimination method with the application to the problem of packaging
Aims of the lesson:
The aim is to raise students' interest in the application of a mathematical method for solving linear equations systems. What is tried to achieve here is students' insight into the meaning and universality of the method and its applicability to solving real problems specific for the problem of packaging set by graphic companies. The lecturer should test students' previous knowledge of mathematics, i.e. linear algebra, which is necessary to understand this lesson, such as matrix calculus, calculation of matrix determinant, finding the inverse of a matrix and the definition of equations system. The problem of packaging should be defined in this part.

Implementation of the lesson
Technique:
Mini-lecture about the mathematical method containing the definition of equations system. Classification of three types of equations systems – equations with a single solution, equations with an infinite number of solutions and systems with no solution. It is necessary to present Kronecker-Capelli theorem and its shortened proof. Thereafter follows the practical part of solving a few system examples on the board and the results are then checked by using an appropriate computer programme. When students master mathematical basics it is necessary to hold a short lecture about the application of this method to the problem of packaging. Therefore the introduction of the example of the problem of packing graphic products which occurs in Kraš factory. This problem is idealized in order to be formulated as a task.

Task (Packaging problem in Kraš factory)
The following problem occurred in Kraš, a chocolate products factory. For transporting biscuits in the volume of 97 cubic meters they have at their disposal 10 large cardboard boxes with the following volumes – 5, 9 and 12 cubic meters. Find the way to transport biscuits.
Solution of the task:
The task is presented as the system of two equations with three unknowns: $x + y + z = 10$ $5x + 9y + 12z = 97$ The obtained system has an infinite number of solutions which can be seen by applying the Kronecker-Capelli theorem. The Gaussian elimination method gives the following result: $\mathrm{x} = -\frac{7}{4} + \frac{3}{4} \cdot \alpha$ $\mathrm{y} = \frac{47}{4} - \frac{7}{4} \cdot \alpha,$ $\mathrm{z} = \alpha, \alpha \in \mathbb{R}$ It is determined in discussion that from the infinite number of theoretically possible solutions of the system, only those solutions should be singled out which make sense within the transport problem. Therefore the following conditions need to be met: $\mathrm{x} \geq 0, \mathrm{y} \geq 0$ i $\mathrm{x}, \mathrm{y} \in \mathbb{N}$ With the help of the direct check it can be determined that the value of parameter α equals $\alpha = 5$. The sought solutions are: $x = 2, y = 3, z = 5$

Interactive discussion between students and the teacher
After students have understood the described procedure it is necessary to start a discussion and take into consideration students' suggestions and proposals so that they learn to recognize and model problems similar to those from the previous task. To be more precise, students should independently cite examples they have learned in graphic technology which in their opinion could be described with the help of equations systems, state what type of equation is in question and solve the system on their own by using the Gaussian elimination method. At the end they should interpret the obtained results and describe the meaning of results as solutions to specific problems from graphic technology.

Testing knowledge
It is necessary to test how much the students have mastered the Gaussian elimination method. Through problem tasks which should be a part of the test, it is necessary to determine have the students gained the capability to set and interpret specific graphic problems with the help of mathematical methods. At the end students should be divided into groups and given assignment to think of problems from graphic discipline which can be described by using equations systems.
Testing technique:
a) Written test b) Presentation of papers

According to the model of the described lesson a large number of mathematics lessons for students of graphic technology should be devised. The teachers of mathematics in cooperation with the scientists from the area of technology should in future work more on gathering real problems to be used in teaching.

CONCLUSION

The 21st century demands interdisciplinarity of universities which includes cooperation between experts from different areas. This opinion is also supported by the Bologna System as well as the American education model. Interdisciplinary approach is also necessary in the teaching of mathematics at Croatian faculties of technology in order to link it with applications from science and technology. This paper gave an example of a lesson at the Faculty of Graphic Arts which is based on cross-curricular linkage of mathematical methods with the application in graphic technology. It is possible to similarly plan also other lessons from mathematics courses at the faculties of technology. This approach increases students' capability to face the current challenges of the modern society so that in the near future graduate engineers and masters of technological sciences could be as competitive as possible at the EU labour market.

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